

GRAVISCALAR DARK MATTER AND SMOOTH GALAXY HALOS

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In the framework of the unimodular metagravity, with the scalar graviton/graviscalar dark matter, a regular anomalous one-parameter solution to the static spherically symmetric metagravity equations in empty space is found. The solution presents a smooth graviscalar halo, with a finite central density profile, qualitatively reproducing the asymptotically flat rotation curves of galaxies. To refine the description studying the axisymmetric case in the presence of luminous matter is in order.

Keywords: Unimodular metagravity; graviscalar dark matter; galaxy halos; density profiles; flat rotation curves.

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1. Introduction

General Relativity (GR) is well-known to be a theory of the massless tensor graviton, with two physical degrees of freedom residing in the metric field. At that, the unphysical metric components are restrained due to the general covariance (GC). In this respect, the unimodular covariance (UC) is a viable alternative to GC. Namely, it has been shown that UC is necessary and sufficient to retain two transverse components for a massless tensor field, with GC being thus excessive to this purpose.¹ This may *de facto* justify all the GC violating alternatives to GR, possessing the residual UC. *Viz.*, they may be considered as the theories of the massless tensor graviton with the different realizations of a metric component corresponding to a dilaton. Retaining this component but making it unphysical due to GC we would arrive at GR. Respectively, two routs to go beyond GR, with the residual UC, are envisaged. *(i)* To eliminate such a component from the metric *ab initio* by means of a unimodularity condition. This would bar the local scale transformations, with the local measure becoming an absolute element.² This is the so-called unimodular relativity/gravity. (For a recent discussion, see, *e.g.*, Refs. 3 – 4.) The cosmological constant emerges here as an integration constant, instead of a Lagrangian parameter in GR. Thereupon, one hopes to naturally explain the cosmological constant being tiny (the long-standing naturalness problem). Furthermore, extending the unimodular relativity/gravity by an exactly massless dilaton one can try to explain the hierarchy problem in SM unified with gravity and simultaneously solve the so-called dark energy problem.⁵ *(ii)* To convert

the aforesaid unphysical (but still “harmless”) component to the physical one by adding to the GR Lagrangian a GC violating term with a derivative of the metric. At that, a (massive) dilaton would arise as a part of the metric field.⁶ Reflecting GC violation, such a route originally implies (a class of) the distinguished, “canonical” coordinates, with the restricted (unimodular) group of the admitted transformations (thereof, the so-called restricted relativity/gravity).

In an earlier paper, we put forward a hypothesis that GC violation with the metric derivative terms may serve as a *raison d’être* for appearance in the Universe of the dark matter (DM) of gravitational origin.⁷ The reason is that under such a GC violation the energy-momentum tensor of the ordinary matter alone ceases to be covariantly conserved. This non-conservation can be compensated by equivalently treating the additional terms in the gravity equations as an energy-momentum tensor for the additional gravity degrees of freedom (the extra “gravitons”) and associating the latter ones with DM. The metric itself serves thus as a resource of DM. As a simplest realization of this approach, the residual UC, with the local scale covariance alone being violated, was imposed. In this case, the metric comprises just one extra physical degree of freedom, a (massive) scalar graviton/graviscalar besides the (massless) tensor graviton. Possessing UC and containing the extra graviton, such a theory may be called the “unimodular metagravity”. By introducing a non-dynamical scalar density we put the theory to the arbitrary observer’s coordinates, beyond the canonical ones. In a subsequent paper, the graviscalar field was taken as an independent variable substituting a metric component in the desired observer’s coordinates.⁸ This allowed us to straightforwardly confront the unimodular metagravity with GR in the presence of an ordinary scalar field. More particularly, an exact “normal” solution to the static spherically symmetric metagravity equations in the empty, but for a singular point, space was written down. The solution is singular in the center and presents the black holes filled with graviscalars. It implies the “normal” rotation curves (RC’s), *i.e.*, those declining asymptotically with distance according to the Newton law.

In the present paper, a regular “anomalous”, missing in GR, solution to the static spherically symmetric metagravity equations in empty space is studied. The solution naturally results in the “anomalous”, asymptotically flat RC’s. It presents a smooth halo as a coherent state of the graviscalar field in the vacuum. Treated in terms of DM the halo possesses a finite central density profile reproducing qualitatively the contribution to the galaxy RC’s due to DM. The way to refine the description of the galaxy halos, as composed of the graviscalar DM, is finally indicated.

2. Anomalous Vacuum Solution

Unimodular metagravity In the framework of the effective field theory of metric, the Lagrangian of the unimodular metagravity looks most generally like:^{7,8}

$$L = L_g + L_h + L_m + L_{gh} + L_{mh}, \quad (1)$$

where the graviton and graviscalar Lagrangians L_g and L_h , respectively, are as follows:

$$L_g = -\left(\frac{\kappa_g^2}{2}R + \Lambda\right), \quad (2)$$

$$L_h = \frac{1}{2} \partial\chi \cdot \partial\chi - V_h(\chi), \quad (3)$$

with χ being the graviscalar field.^a In the above, $\kappa_g = 1/(8\pi G)^{1/2}$ is the GR mass scale, with G standing for the Newton's constant, R is the Ricci scalar, Λ is the cosmological constant, V_h is the graviscalar potential and $\partial\chi \cdot \partial\chi = g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi$, with $g_{\mu\nu}$ being the metric. The Lagrangian L_m for an ordinary matter has some conventional form. In the minimal metagravity, we consider, $L_{gh} = L_{mh} = 0$.

The peculiarity of the graviscalar compared to an ordinary scalar is that the former is not independent of the metric, *viz.*,

$$\chi = \frac{\kappa_h}{2} \ln \frac{g}{g_h}, \quad (4)$$

where $g = \det g_{\mu\nu}$ and g_h is a non-dynamical scalar density of the same weight as g . In the canonical coordinates, we have $g_h = -1$. The density g_h makes χ a GC scalar and allows to bring the theory to the arbitrary observer's coordinates. The parameter κ_h stands for a unimodular metagravity mass scale additional to the GR κ_g . Presumably, $\kappa_h \leq \mathcal{O}(\kappa_g)$.

Varying the action $S = \int d^4x \sqrt{-g} L$ with respect to $g_{\mu\nu}$, under fixed g_h , we arrive at the unimodular metagravity equations as follows

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{1}{\kappa_g^2} T_{\mu\nu}, \quad T_{\mu\nu} = T_{\Lambda\mu\nu} + T_{m\mu\nu} + T_{h\mu\nu}, \quad (5)$$

with $R_{\mu\nu}$ being the Ricci curvature tensor. In the above, $T_{\Lambda\mu\nu} = \Lambda g_{\mu\nu}$ is the vacuum contribution to the total energy-momentum tensor $T_{\mu\nu}$, with $T_{m\mu\nu}$ being the ordinary matter contribution and $T_{h\mu\nu}$ the graviscalar one. The latter looks like

$$T_{h\mu\nu} = \partial_\mu \chi \partial_\nu \chi - \left(\frac{1}{2} \partial\chi \cdot \partial\chi - \check{V}_h \right) g_{\mu\nu}, \quad (6)$$

where

$$\check{V}_h = V_h + \kappa_h \left(\partial V_h / \partial \chi + \nabla \cdot \nabla \chi \right), \quad (7)$$

with ∇_μ standing for a covariant derivative. The contracted Bianchi identity, $\nabla_\mu G^\mu_\nu = 0$, results in the covariant conservation of the total energy-momentum, $\nabla_\mu T^\mu_\nu = 0$, instead of $\nabla_\mu T^\mu_\nu = 0$ for the ordinary matter alone. (Thereof, the treatment of the graviscalar as DM). Eq. (6) resembles that for an ordinary scalar field in GR except for the metapotential \check{V}_h superseding the conventional potential V_h . When dealing with the metagravity equations, we can proceed in the canonical coordinates, $g_h = -1$, followed by a transformation to the observer's coordinates x^μ . Instead, we proceed directly in x^μ , with χ taken as an independent variable, which substitutes a metric element fixed by an additional coordinate condition. At that, the unknown g_h does not enter the calculations explicitly. Having found metric and χ we can then through Eq. (4) recover in the same coordinates the required g_h , solving in a sense an inverse problem.

^aThe graviscalar being a kind of a "hidden" particle, the related quantities are endowed with a subscript h.

In what follows, we restrict ourselves to empty space, $T_{m\mu\nu} = 0$. In this case, the contracted Bianchi identity results in the graviscalar field equation as follows:⁸

$$\nabla \cdot \nabla \chi + \partial \check{V}_h / \partial \chi = 0, \quad (8)$$

with \check{V}_h reduced to

$$\check{V}_h = V_h - w_h e^{-\chi/\kappa_h}. \quad (9)$$

Here, w_h is an arbitrary integration constant, not a Lagrangian parameter, which distinguishes the local vacua.

Particularly, consider the static spherically symmetric configuration of metric and the graviscalar field. The line element in the polar coordinates (t, r, θ, φ) looks generally like

$$ds^2 = adt^2 - bdr^2 - cr^2 d\Omega, \quad d\Omega = d\theta^2 + \sin^2 \theta d\varphi^2, \quad (10)$$

with the three metric potentials a , b and c depending on the radial coordinate r alone. The same is supposed about χ . Specify r by the coordinate condition $ab = 1$ and choose χ as the third independent variable instead of b . Neglect by the potential V_h and the cosmological constant Λ . Putting $X = \chi/\kappa_h$, $A = a = 1/b$ and $C = r^2 c$, we get Eq. (8) as

$$(ACX')' = \frac{w_h}{\kappa_h^2} C e^{-X}, \quad (11)$$

with a prime meaning a derivative with respect to r , and the unimodular metagravity equations in the vacuum as⁸

$$(CA')' = \frac{2w_h}{\kappa_g^2} C e^{-X}, \quad (12)$$

$$(CC')' - \frac{3}{2} C'^2 = -\frac{\kappa_h^2}{\kappa_g^2} (CX')^2, \quad (13)$$

$$(CA')' - (AC')' + 2 = 0. \quad (14)$$

By construction, Eqs. (12) - (14) are independent, with Eq. (11) being an identity. Instead, we choose Eqs. (11) - (13) as the independent ones, with Eq. (14) serving as a constraint.

Anomalous vacuum solution Let first κ_h be arbitrary, $\kappa_h \leq \mathcal{O}(\kappa_g)$. At $w_h = 0$, an exact solution to the metagravity equations was given in Ref. 8. At $r = 0$, the solution is singular (reflecting a center point-like matter). This case corresponds to GR in the presence of a scalar field. With $w_h \neq 0$, there appears a solution regular at $r = 0$. Expanding the unknown functions as the power series in r and equating coefficients at equal powers on both sides of equations we have up to terms r^6 :

$$X = \tau^2 - \frac{1}{2} \left(\frac{3}{5} + \varepsilon_h^2 \right) \tau^4 + \left(\frac{1}{35} \left(4 + \frac{41}{3} \varepsilon_h^2 \right) + \frac{1}{3} \varepsilon_h^4 \right) \tau^6, \quad (15)$$

$$a - 1 = \varepsilon_h^2 \left(\tau^2 - \frac{3}{10} \tau^4 + \frac{1}{35} \left(4 + \frac{19}{6} \varepsilon_h^2 \right) \tau^6 \right), \quad (16)$$

$$c - 1 = \varepsilon_h^2 \left(-\frac{1}{10} \tau^4 + \frac{2}{7} \left(\frac{1}{5} + \frac{1}{3} \varepsilon_h^2 \right) \tau^6 \right), \quad (17)$$

where $\varepsilon_h^2 = 2\kappa_h^2/\kappa_g^2$ and $\tau^2 = r^2/R_h^2$, with $R_h^2 = 6\kappa_h^2/w_h$ presenting a characteristic length scale squared. Eq. (14) is fulfilled identically up to terms τ^6 .

Continuing the procedure above we can find the solution with any desired accuracy. Namely, knowing the solution in an order τ^{2n} , $n = 0, 1, 2, \dots$ we can first determine X from the l.h.s. of Eq. (11) in the next order $\tau^{2(n+1)}$. Then we can find a and c in the same order from Eqs. (12) and (13), respectively, *etc.* At that, $X(0)$, $a(0)$ and $c(0)$ are fixed by the boundary conditions at $\tau = 0$. Both $w_h > 0$ and $w_h < 0$ are *a priori* envisaged. The respective solutions are formally related by substitution $\tau^2 \rightarrow -\tau^2$. For physical reasons, $w_h \geq 0$ (see later).

Of special interest is the case $\varepsilon_h \ll 1$. Decomposing an exact solution, regular at the center, as the power series in ε_h (only even powers enter) as $X = \sum \varepsilon_h^{2n} X_n$, $a = \sum \varepsilon_h^{2n} a_n$ and $c = \sum \varepsilon_h^{2n} c_n$, $n = 0, 1, \dots$, with $a_0 = c_0 = 1$, we simplify the metagravity equations in the appropriate leading orders as follows:

$$\frac{d}{d\tau} \left(\tau^2 \frac{dX_0}{d\tau} \right) = 6\tau^2 e^{-X_0}, \quad (18)$$

$$\frac{d}{d\tau} \left(\tau^2 \frac{da_1}{d\tau} \right) = 6\tau^2 e^{-X_0}, \quad (19)$$

$$\frac{d}{d\tau} \left(\tau^2 \frac{dc_1}{d\tau} \right) = -\frac{1}{2} \left(\tau \frac{dX_0}{d\tau} \right)^2, \quad (20)$$

with the restriction

$$\frac{d}{d\tau} \left(\tau^2 \frac{d}{d\tau} (a_1 - c_1) - 2\tau(a_1 + c_1) \right) = 0. \quad (21)$$

Clearly, it is possible to add to the solutions for a_1 and c_1 the arbitrary reciprocal terms $\sim 1/\tau$. Assuming no singularity in the center, we omit such contributions. It follows from the equations above that the driving term in the system is X_0 . Having found the latter in a self-consistent manner from Eq. (18) we can then find a_1 and c_1 from the two other equations with an external source determined by X_0 . In particular, it follows that $a_1 = X_0$ modulo a constant which may be put to zero. The leading in ε_h part of the regular solution given by Eqs. (15) – (17) explicitly satisfies all these equations up to accuracy τ^6 .

To study X_0 at $\tau^2 \geq 0$ *in toto* note first of all that there exists an exact exceptional solution of Eqs. (18) – (20) as follows:

$$\bar{X}_0 = \ln 3\tau^2, \quad (22)$$

$$\bar{a}_1 = \ln 3\tau^2, \quad (23)$$

$$\bar{c}_1 = -\ln 3\tau^2 + 2, \quad (24)$$

with the additive constants restricted by the relation $\bar{a}_1 = \bar{X}_0$ and Eq. (21). Present further Eq. (18) as follows:

$$\frac{d^2 Z}{d\sigma^2} + \frac{1}{2} \frac{dZ}{d\sigma} = \frac{1}{2} (e^{-Z} - 1) \quad (25)$$

where $Z = X_0 - \sigma$, with $\sigma = \ln 3\tau^2$ any real, $-\infty < \sigma < +\infty$. Introducing $\dot{Z} \equiv dZ/d\sigma$ as an independent variable supplementing Z , reduce the second-order Eq. (25) to the

equivalent autonomous first-order system:

$$\frac{dZ}{d\sigma} = \dot{Z}, \quad (26)$$

$$\frac{d\dot{Z}}{d\sigma} = -\frac{1}{2}\dot{Z} + \frac{1}{2}(e^{-Z} - 1). \quad (27)$$

In the phase plane (Z, \dot{Z}) , there is a single exceptional point $\bar{Z} = \bar{\dot{Z}} = 0$, defined by the requirement $dZ/d\sigma = d\dot{Z}/d\sigma = 0$, other points being normal. Through each normal point there should come precisely one phase trajectory $(Z(\sigma), \dot{Z}(\sigma))$. The latter ones satisfy the equation

$$\frac{d\dot{Z}}{dZ} = \frac{1}{2\dot{Z}}(e^{-Z} - 1) - \frac{1}{2}, \quad (28)$$

with the isoclines $d\dot{Z}/dZ = m$ being

$$\dot{Z} = \frac{1}{2m+1}(e^{-Z} - 1), \quad (29)$$

where m is an arbitrary constant. At that, the axes $\dot{Z} = 0$ and $Z = 0$ correspond to $m \rightarrow \pm\infty$ and $m = -1/2$, respectively.

Inspection of the phase plane shows that the exceptional point in the center belongs to the stable focus type, with all the trajectories winding round the center and approaching the latter with $\sigma \rightarrow +\infty$. At that, the exceptional point presents the exceptional solution Eq. (22). There is a unique trajectory with \dot{Z} remaining finite at $\sigma \rightarrow -\infty$, namely, $\dot{Z} \rightarrow -1$, and behaving thus like $Z \simeq -\sigma$ asymptotically. Such a trajectory corresponds to the regular at $\tau = 0$ solution X_0 given by Eq. (15) with $\varepsilon_h = 0$. The rest of trajectories satisfy $\dot{Z} \rightarrow -\infty$ at $\sigma \rightarrow -\infty$, with the respective X_0 being thus irregular at $\tau = 0$. The regular anomalous solution is stable against small perturbations of the initial data taken on the axis $\dot{Z} = 0$, $Z < 0$, but for very small $\tau^2 > 0$. In the latter region, the regular anomalous solution is to be superseded by a singular normal solution with $r_h \ll R_h$, where r_h is the graviscalar radius of a center singularity.⁸ The account for the latter does not significantly affect the halo at $\tau \gg r_h/R_h$.

Altogether, the regular anomalous vacuum solution for the graviscalar field X at $\varepsilon_h \ll 1$ looks like:

$$X_0 = \begin{cases} \tau^2 - \frac{3}{10}\tau^4 + \frac{4}{35}\tau^6 + \mathcal{O}(\tau^8), & \text{at } 0 \leq \tau < 1, \\ \ln 3\tau^2, & \text{at } \tau \gg 1. \end{cases} \quad (30)$$

It oscillates around the exceptional solution \bar{X}_0 approaching the latter at $\tau \gg 1$.

3. Anomalous Rotation Curves

Graviscalar DM The velocity of circular rotation of a test particle in the static spherically symmetric metric Eq. (10) is given by

$$v^2 = \frac{a'}{(\ln r^2 c)'} \quad (31)$$

So defined velocity transforms as a scalar under the local radial transformations. To get v^2 in the leading ε_h -order we can put $c = c_0 = 1$. At $w_h > 0$, the regular anomalous solution results in the scaled RC profile as follows:

$$v_h^2(\tau) = \frac{\varepsilon_h^2}{2} \frac{\tau da_1}{d\tau} = \varepsilon_h^2 \begin{cases} \tau^2 - \frac{3}{5}\tau^4 + \frac{12}{35}\tau^6 + \mathcal{O}(\tau^8), & \text{at } 0 \leq \tau < 1, \\ 1, & \text{at } \tau \gg 1, \end{cases} \quad (32)$$

where use is made of $a_1 = X_0$. At that, the exceptional solution results in the flat RC

$$\bar{v}_h^2(r) = \varepsilon_h^2, \quad (33)$$

around which all the RC's $v_h^2(r)$, with different R_h , oscillate approaching \bar{v}_h^2 at $r \gg R_h$.

Let us now interpret RC's in terms of DM. The Newton dynamics in flat space ($a = c = 1$) with a DM would result in

$$\frac{v_h^2}{r} = \frac{GM_h(r)}{r^2}, \quad (34)$$

where $M_h(r) = 4\pi \int_0^r \rho_h(r') r'^2 dr'$ is the DM energy interior to r , with ρ_h being the DM energy density. This implies

$$\rho_h = \frac{1}{4\pi G} \frac{(rv_h^2)'}{r^2}. \quad (35)$$

To reproduce the first part of Eq. (32) we should have

$$\rho_h = \frac{\varepsilon_h^2 \kappa_g^2}{R_h^2} \frac{1}{\tau^2} \frac{d}{d\tau} \left(\tau^2 \frac{da_1}{d\tau} \right). \quad (36)$$

(For a center point-like matter with $a - 1 \sim -1/r$ and $v^2 \sim 1/r$, this would give $\rho_h = 0$.) Accounting for Eq. (19), we get finally the looked-for DM profile as follows ($\tau^2 \geq 0$):

$$\rho_h(\tau) = 2w_h e^{-X_0} = \rho_h(0) \begin{cases} 1 - \tau^2 + \frac{4}{5}\tau^4 + \mathcal{O}(\tau^6), & \text{at } 0 \leq \tau < 1, \\ 1/(3\tau^2), & \text{at } \tau \gg 1, \end{cases} \quad (37)$$

with the central density

$$\rho_h(0) = \frac{6\varepsilon_h^2 \kappa_g^2}{R_h^2}. \quad (38)$$

Asymptotically, $M_h(r) \simeq \varepsilon_h^2 r/G$. Ultimately, such a linear growth should be terminated by the potential V_h , which would become significant at the periphery, where X_0 gets strong. Thus the regular anomalous solution corresponds to a smooth DM halo with the finite central density. At that, the exceptional solution results in the cuspy profile

$$\bar{\rho}_h(r) = \frac{2\varepsilon_h^2 \kappa_g^2}{r^2}, \quad (39)$$

with the exact $\bar{M}_h(r) = \varepsilon_h^2 r/G$. The family of the smooth profiles $\rho_h(r)$, with various R_h , oscillates around $\bar{\rho}_h$ approaching the latter at $r \gg R_h$.

According to Ref. 8, $\rho_h = -2\tilde{V}_h$ may be treated as the energy density of a static graviscalar field, incorporating its gravitational energy. This insures a dual field-matter

interpretation of the graviscalar halo. In terms of field, the case $w_h > 0$ presents a local vacuum well, with the metapotential $\check{V}_h = -w_h e^{-X_0}$ due to a coherent state X_0 of the graviscalar field. In terms of matter, the same case corresponds to the DM distribution with $\rho_h > 0$ which produces in flat space precisely the same attraction. The case $w_h < 0$ presents a local vacuum bump with the repulsive $\rho_h < 0$ implying an unstable configuration ($v_h^2 < 0$). In the GR limit, $w_h = 0$, the halo clearly disappears.

Galaxy halos There are numerous studies in astrophysical literature concerning the galaxy DM halos. At that, the empirical halo density profiles rely mostly on the two-component fits to the galaxy RC's, with the matter and halo contributions added in quadrature, $v^2 = v_m^2 + v_h^2$, where v_m^2 is a total contribution of the different types of luminous matter (disk, gas, bulge) and v_h^2 is a halo contribution. Thereof, there emerge ever growing evidences, based on a vast sample of galaxies of different types, in favour of the DM halos with the finite central density profiles (see, *e.g.*, Refs. 9 – 11, with an extensive list of references therein). In particular, in Ref. 10 it is found a universal DM density profile, extracted from a sample of 36 nearby spiral galaxies, as follows:

$$\rho_h = \frac{\rho_0}{1 + (r/R_0)^2} \quad (40)$$

with ρ_0 being a central density and R_0 a core radius. The empirical smoothness displayed by Eq. (40) is at sharp variance with the cuspy form of the cold DM halos. In contrast, the vacuum graviscalar halo obtained in the present paper naturally complies with smoothness. Moreover, Eq. (37) closely reproduces the first three terms of the decomposition of Eq. (40). Nevertheless, there are two differences. First, Eq. (38) implies $\rho_h(0) \sim R_h^{-2}$, whereas empirically there emerges a constant central surface density of the galaxy halos, *i.e.*, $\rho_0 R_0 \sim \text{const.}$ Second, Eq. (37) is three times lower asymptotically compared to Eq. (40).

The reason of the discrepancy may be as follows. We have restricted ourselves by the simplest model with the spherical graviscalar halo in empty space. Such a halo may serve just as a prototype for the real galaxies. To confront the theory with the data the luminous matter should also be accounted for. This would give $v^2 = v_m^2 + v_{hm}^2$, with v_{hm}^2 being the effective graviscalar halo contribution in the presence of matter. Because of a coherent nature of halo the deformation of the latter, both in magnitude and sphericity, may be significant within the region of intersection of matter and halo. It is rather v_{hm}^2 , to which the two-component fit Eq. (40) is to be applied, than v_h^2 due the vacuum graviscalar halo. To distil the galaxy RC sample from the matter contribution as far as possible, live aside the points from Ref. 10 which correspond explicitly to the luminous matter dominance at the distances at hand. Inspection shows that there are at least four, out of 36, points on the $\log \rho_0 - \log R_0$ plot, with the extremely large R_0 , to be dropped off. (Incidentally, such peculiar R_0 are strongly model dependent.) The rest of points lies much more compactly, the subsequent results being less sensitive to further reducing the galaxy sample. On the reduced sample, the dependence $\log \rho_0 \sim -\log R_0$ looks less prominent. On the other hand, a much wider sample of galaxies of different types still supports the constant central surface density law.^{11,b} To settle the question in the metagravity framework the account

^bThe latter data may also indicate some deviations from Eq. (40) both at very small and very large

for the luminous matter, with an axisymmetric distribution resulting in the graviscalar halo asphericity, is required.

Finally note that it is the Lagrangian parameter $\varepsilon_h = \sqrt{2}\kappa_h/\kappa_g$, which sets the scale of the asymptotically flat RC's due to the graviscalar halo in the vacuum. So, taking for galaxies asymptotically $v_h(\infty) \sim 100$ km/s we would expect that $\varepsilon_h = v_h(\infty)/c \sim 10^{-3}$. With $\kappa_g = 2.4 \times 10^{18}$ GeV, the unimodular metagravity mass scale, $\kappa_h \sim 10^{15}$ GeV, would approach the GUT mass scale.

4. Conclusion

The regular anomalous solution to the static spherically symmetric metagravity equations in empty space presents a viable prototype model for the smooth galaxy halos characterized by the finite central density profiles. It goes without saying that once the solution models the descent density profiles of the DM halos, it provides to the same extent all the other gravitational effects of such halos. The hypothesis about the graviscalar origin of DM in the framework of the unimodular metagravity finds thus its preliminary confirmation. To further verify the theory studying the graviscalar halos in the presence of the axisymmetric matter distribution is in order.

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